Basics in linear algebra and numerical methods

Instructor:Imre Fekete

Term: Fall

Weeks: 1-7

Contact hours: 3

Credits: 6

Aim and scope:

A majority of operations in computer programs for machine learning can be recognized as matrix - vector operations.

Its mathematical background is linear algebra, the basics of which are summarized in this course. Additionally, common numerical methods are also presented in this framework.

The theoretical background is supplemented by weekly practices in Python.

Syllabus:

Linear algebra refreshing: vectors, matrices, basic operations

(flipping, concatenating, cancelling, sorting ...)

Matrices everywhere.

Representation, partitions - equivalence relations.

Geometrical interpretation. Subspaces, range of matrices. Projections.

Special matrices - continued: projections, rotation, permutations.

Connection with matrix multiplications.

Need for sparse matrices, operations with these.

Linear systems, their numerical solution, number of operations.

Eigenvalue problems, real and complex eigenvalues. Eigenvalues of stochastic matrices.

Numerical methods for finding eigenvalues: power method, eigenvalue decompositions, SVD decomposition.

Random variables, expected value, variance. Random vectors, random matrices.

Mean and matrices, moving average, standardization of data.

Distance: metrics and norms in connection with losses. Important examples.

Loss minimization: extrema of multivariable functions with linear algebra. Connection with convex functions. Example: linear regression.

The gradient method I : discussion of initialization and parameters.

Stochastic gradient methods: advance and explanation of the parameters.

Practical computations with Python.

Grading: exam (incorporating the solution of homeworks)

Literature:

L.N. Trefethen and D. Bau: Numerical Linear Algebra, SIAM Publications, Philadelphia, PA, 1997.

H.P. Langtangen: A Primer on Scientific Programming with Python, 3rd ed., Springer, 2012

L.N. Trefethen and D. Bau III (1997). *Numerical Linear Algebra*, SIAM Publications, Philadelphia, PA, 1997.