

Optimisation

Instructor: Kristóf Bérczi

Term: Fall

Weeks: 1-7

Contact hours: 3

Credits: 6

Aim and scope:

Knowledge of basics of combinatorial optimisation, including elements of linear programming.

Understanding the main ideas of LP solving techniques.

Ability to recognize convex programming problems, knowledge of gradient descent algorithm.

The theoretical background is supplemented by weekly practices in Python.

Prerequisites: calculus, linear algebra, matrices and eigenvalues

Syllabus:

Basics of linear programming: linear equations, Gauss elimination, Fredholm alternative theorem, polyhedra, polytopes. Linear inequalities: basic feasible solutions, Farkas lemma, Duality theorem. Simplex algorithm: general idea, performance, variants

Heuristics: branch and bound, dynamic programming, local search, simulated annealing

Convexity and optimization: convex sets, convex functions, convex optimization. Duality: Lagrangian dual, conjugate function.

Kuhn-Tucker theorem.

Gradient descent: general idea, Lipschitz gradient. Mirror descent and multiplicative weight update method: regularizers, exponential gradient descent, multiplicative weights update framework.

Stochastic gradient descent: background, iterative method, extensions.

If time permits: Interior point methods: affine scaling, potential reduction, path following methods, large scale optimization: column generation, cutting plane methods.

Grading: term mark (incorporating the solution of homeworks)

Literature:

András Frank. Connections in combinatorial optimization. Oxford Lecture Series in Mathematics and Its Applications 38, 2012.

Dimitris Bertsimas, John N. Tsitsiklis. Introduction to linear optimization. Belmont, MA: Athena Scientific, 1997.

Nisheeth Vishnoi. Algorithms for Convex Optimization.